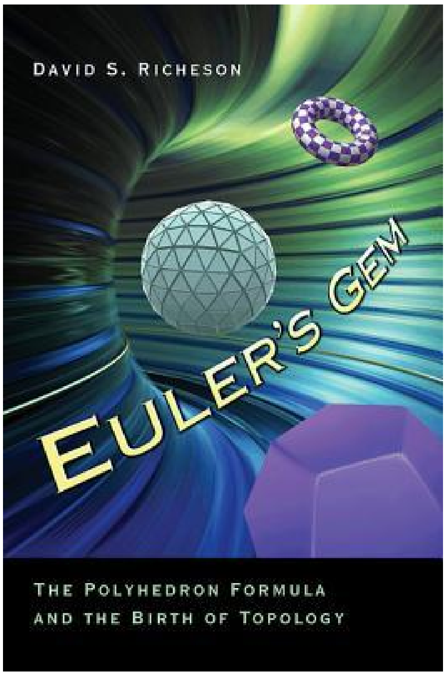

Euler's Gem: The Polyhedron Formula and the Birth of Topology

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Leonhard Euler's polyhedron formula describes the structure of many objects--from soccer balls and gemstones to Buckminster Fuller's buildings and giant all-carbon molecules. Yet Euler's formula is so simple it can be explained to a child. *Euler's Gem* tells the illuminating story of this indispensable mathematical idea.

From ancient Greek geometry to today's cutting-edge research, Euler's Gem celebrates the discovery of Euler's beloved polyhedron formula and its far-reaching impact on topology, the study of shapes. In 1750, Euler observed that any polyhedron composed of V vertices, E edges, and F faces satisfies the equation V-E+F=2. David Richeson tells how the Greeks missed the formula entirely; how Descartes almost discovered it but fell short; how nineteenth-century mathematicians widened the formula's scope in ways that Euler never envisioned by adapting it for use with doughnut shapes, smooth surfaces, and higher dimensional shapes; and how twentieth-century mathematicians discovered that every shape has its own Euler's formula. Using wonderful examples and numerous illustrations, Richeson presents the formula's many elegant and unexpected applications, such as showing why there is always some windless spot on earth, how to measure the acreage of a tree farm by counting trees, and how many crayons are needed to color any map.

Filled with a who's who of brilliant mathematicians who questioned, refined, and contributed to a remarkable theorem's development, *Euler's Gem* will fascinate every mathematics enthusiast.

Euler's Gem: The Polyhedron Formula and the Birth of Topology Details

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From Reader Review Euler's Gem: The Polyhedron Formula and the Birth of Topology for online ebook

Bob Gustafson says

Richeson's "Euler's Gem" is an excellent book. It gives the historical background, going back to ancient Greece, for this equation regarding faces, edges and vertices of polyhedra. It tells us about Euler (as well as more than a dozen other mathematical scholars) and the relationship. It goes on to tell us about various proofs and then extensions of and enhancements to the equation.

If this book is excellent, why four rather than five stars? It's because of the medium. This book was brought to my attention by "The Shape of Nature" a lecture series by Professor Devados on DVD from The Great Courses. Drawings in two-space can only do so much. Dynamic demonstrations in three-space, even though the computer screen is flat, makes things so much clearer.

An excellent presentation on DVD is simply better than an excellent book.

Aleksandr Jermakov says

"The scientist does not study nature because it is useful; he studies it because he delights in it, and he delights in it because it is beautiful. If nature were not beautiful, it would not be worth knowing, and if nature were not worth knowing, life would not be worth living."

These are Poincaré's words author quotes at the end of the book. And he very well manages to show the nature's beauty in his book. More of biography/history of science than actual Mathematical technicalities so if you are into that sort of thing might want to give it a go.

Ruth says

I read this book on the history of topology to my elementary-school-aged son. I cannot really remember when we finished it, but it was some time in the summer of 2011. The idea of a book for a popular audience about topology is absurd. Once I owned a book called *Learn Gujrati in 30 Days*, and this was something like that. My kid was too young to realize that it was hard to understand and just had a lot of fun. We read about a chapter a night for weeks and weeks. At the very end of the book was a chapter about Emmy Noether, which made me very happy. I didn't understand the math she did, either, but I liked knowing more about her.

Adam says

This turned out to be an excellent introduction to topology via Euler's second most famous formula, $V - E + F = 2$. The first 30 pages were a history lesson, and after that it kept ramping up the math, ending at about the same place where my brain stopped working in grad school (homology groups). The proofs are mostly of the "general overview without the nasty tricky details" variety, which is about right for a book like this.

Recommended for those who want a pop-math introduction to topology or for former math majors who like to be reminded about how much they've forgotten.

Matthew Dambro says

Excellent introduction to topology. Much of the math was over my head but the story of how mathematics is done is superb. The step by step accumulation of knowledge with the occasional flash of brilliance shown by a Euler or a Poincare is breathtakingly beautiful.

Daniela says

I am more or less in the middle of this book and I'm loving it. It explains, with simple words, the relevance of the polyhedron formula; filled up with history and proofs. Thumbs up!

Mohammed Hashem says

Great recap of the history of the relatively new mathematical field of Topology.

A quote I liked...

“Scientists and engineers have used computers to solve countless problems, but mathematicians have not. Computers are good at making speedy calculations, but not at the kind of precise and subtle arguments that are required in mathematical proof. Like philosophy and art, mathematics has always been a human endeavor, one that cannot be automated.” David S. Richeson, Euler’s Gem

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Upom says

I've been dreaming of higher dimensions lately, and this book on topology just enthralled this fascination even more. "Euler's Gem" is really a look at one of the most famous equations you've never heard: V-E+F=2, also known as Euler's Formula. This formula originally described a relationship between the faces, edges, and vertices of the 5 platonic Solids, but actually has a deeper significance as an equation connecting the vast subtopics of topology, including graph theory, knot theory, the 4-colors problem, dynamic systems, and homology. Richeson uses Euler's Formula as a way to present the history of toplogy, as well as a grand tour of this little-known topic. Speckled with nice illustrations, pop-culture references, and some really crisp writing, Dickeson reveals a subject that radiates with wonder. The proofs are at times a bit hard to follow, but with a little patience and periodic page-flipping, they are easy enough to follow. The book also contains a nice little appendix with cutouts that you can use to make your own platonic solids, toruses (torii?), Klein bottle, and projective planes. The book could have done with a glossary, and I found myself reading proofs again and again to make sense of them. However, if you give it a little time and effort, "Euler's Gem" is a book that will blow your mind. Just as the book reminds us in the last line with Poincare's words, "The scientist does not study nature because it is useful; he studies it because he delights in it, and he delights in it because it is beautiful."

Koen Crolla says

Euler has arguably produced a lot of gems, but the one Richeson is talking about is the observation that for convex polyhedra, the number of vertices minus the number of edges plus the number of faces is always equal to 2. That this is not true for less sensible polyhedra (it's 0 for a reasonable polyhedral torus, for example) is one of the foundational observations of the field of topology, which is indeed what the book is about.

Euler's Gem is more or less what popular mathematics should be: it's gentle enough for a complete lay audience to follow from start to finish, but also thorough and not afraid to get technical. It covers some history, but doesn't repeat popular myths or devolve into crass gossip. The title isn't necessarily ideal—there are some other, largely unrelated observations at the basis of the field, which Richeson also covers and which fit the historical narrative, but not the title's conceit—and the cover art is awful enough that I was seriously considering covering it up, but on the whole, it's definitely one of the better general-audience books on mathematics I've read.

Trinity School Summer Reading says

Leonhard Euler was an incredibly prolific mathematician who lived in the 18th Century. Among his many fabulous creations, his polyhedron formula is one that describes the structure of 3D shapes from soccer balls to salt crystals. In this book, Richeson describes the development of this indispensable mathematical idea and its connection to the modern fields of graph theory and topology.

Gabriel Daleson says

The first two thirds of this book is heavier on the history and geometry, and it's the better part. The back third gets into topology, and, while it's certainly interesting, it's also not for the squeamish. I'm really not sure how one could easily provide a casual treatment of any high-dimensional mathematics, even as casual as Richeson does here, without losing people. I appreciate the effort, certainly, but I can make a good argument that the chapters on the higher-dimensional analog to the Euler formula should just have been stripped out.

All told, though, it's great to have a book like this, and I hope it gets some promising high school juniors and seniors, and a few new college freshmen, thinking about mathematics and topology.

Erickson says

Nice topology and geometry book. The proof part is especially good. Would recommend this to go along with Weeks' The Shape of Space, or vice versa.

Richard Holmes says

This book wasn't what I was expecting; rather than a book centered on Euler's formula, it's an introduction to topology with the formula as a recurring theme. As such large parts of the book serve to introduce topics having little to do with the formula and which I was already familiar with. I didn't find Richeson's pedestrian exposition shed much in the way of interesting new light on these topics for the most part. I ended up skipping a lot, because it just wasn't what I was looking for.

Matthewmartinmurray murray says

This was really fun to read at first but then goes heavily into topology and left me in the dust a little. I have a better understanding of some topological ideas but could have used a better explanation of most. I am much more comfortable visualizing the projective plan now which I am glad for. I also was interested in seeing some knot theory tie in. Its a good insight to one of the weirder sides of mathematics. So it was fairly good.

Charles says

A gem of mathematical results produced by one of the masters of mathematics

The title of the book is derived from the formula V - E + F = 2 that holds for any convex polyhedron. V is the number of vertices, E the number of edges and F the number of faces. First demonstrated by Euler, the proof of this result is surprisingly simple. As is the case with most such formulas and their proofs, there is at least one near miss in the history of mathematics. Descartes was close; in retrospect it is somewhat surprising that he didn't reach the appropriate conclusion. Of course, we are considering the great master Euler here, a giant of mathematics who was able to see things in his mathematical sight that people with the physical vision that he lacked overlooked.
Topology is a relatively recent area of mathematics, one of the few that can be considered to have had a point of origin and a creator. Richison works through the historical mathematical preliminaries of the formula, the shapes it describes were well known to the ancient Greeks yet they were nowhere close to the formula. Some historical and mathematical background on Euler follows this and it includes some of his other accomplishments. The last chapters describe some of the results that follow from topology in general and Euler's gem in particular. One of the most interesting is the theorem of combing a sphere, where the conclusion is that there must always be at least one hair that stands straight up. This may seem like an absurd thing for mathematicians to be concerned about but it has a major conclusion, that at all times there must be at least one point on Earth where there is no wind. Even more significantly it means that there will always be a zero.
Richison uses a large number of diagrams and formulas when needed, which is to his credit. Mathematics is based on equations so when an author deliberately avoids them in an attempt to increase sales, it is hard to claim that they are actually writing mathematics. This is an excellent book about a great man and a timeless formula. Well within the reach of the intelligent layperson, it is also a good book to use as a resource for a course where the students are required to make presentations.

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